Classical graphs decomposition and their totally decomposable graphs

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Summary
In solving optimization problems on graphs, graph decomposition is considered to be a powerful tool for obtaining efficient solutions for these problems. Totally decomposable graphs with respect to some method of graphs decomposition are relevant to many different areas of applied mathematics and computer science. There is a considerable number of results in this area. The goal of this paper is to survey the state of art of the famous methods of graphs decomposition and their totally decomposable graphs.

Key words:
Graph Problems, Complexity

1. Introduction

In several domains of scientific activities, the efficient solution of a problem P is obtained by dividing the initial problem into sub-problems, then finding a solution for each one and finally constructing a solution for the initial problem from the obtained solutions of these sub-problems.

When a problem P is represented by a graph G, the above procedure can be used as follows: First, divide G into a set H of subgraphs Then apply a solution for each sub-graph in H and finally deduce the solution of P from the obtained solutions of each graph in H. Frequently the set H can be obtained by applying recursively to G one or more operators of decomposition.
Thus, the existence and the efficiency of a solution of problem P using the previous procedure depend on three issues:

1. Is it possible to decompose G efficiently into a set H of subgraphs? We recall that several problems of graphs decomposition are NP-complete (partition into forests, cliques, isomorphic subgraphs etc [GJ79]).
2. What is the complexity of a solution of P for a graph of H?
3. How and with which complexity can we deduce the solution of P for G from the given solutions of subgraph in H?

A popular approach to graph decomposition involves associating with a given graph G a rooted tree $T(G)$ whose internal nodes correspond to the operators of decomposition that are applied and whose leaves correspond to certain subgraphs of G (e.g. vertices, edges, cliques, stable sets, cutsets). Of a particular interest are graphs for which the following conditions hold :

a) $T(G)$ can be obtained efficiently, that is, in time polynomial in the size of G;
b) The tree $T(G)$ is unique (up to isomorphism). This exclusively will permit to transfer the difficulty of recognition problem of G to the verification of a set of properties that concern $T(G)$.

Tree representation satisfying the conditions mentioned above have been obtained for several classes of graphs including the cographs [CPS85], interval graphs [BL79], chordal graphs [Gol80], maximal outerplaner graphs [BJM78], P_4- reducible graphs [JO89], P_4-extendible graphs [JO91], P_4-sparse graphs [JO92], among many others. One of the best exponents of graphs are those that are totally decomposable. A graph G is totally decomposable with respect to the operators of decomposition that are used if it can be decomposed to its vertices by a finite sequence of some of these operators. In this case the set of leaves of $T(G)$ is the set of vertices of G. In the majority of cases we consider that a problem P has a trivial solution for the singletons, consequently the difficulty of the solution of P is subject to points 1 and 3 above.

The purpose of this paper is to present the state of the art about the most known methods of graph decomposition and the classes of graphs that are totally decomposable with respect to these methods.
The paper is organized as follows : In the rest of this section, we are giving the basic concepts to be used throughout this paper. Section 2 presents the modular decomposition and cographs.. Section 3 presents the homogeneous decomposition, P_4-reducible graphs and P_4-sparse graphs. Section 4 presents split decomposition and completely separable graphs.

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